**Problema 1**

a) Falso, por ejemplo la matriz $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ tiene valores propios nulos, a pesar de ser una matriz no nula.

b)

```
% silvester.m
function [dp]=silvester(A)
n=size(A,1);
dp=1;
for k=1:n
   m=A(1:k,1:k);
   if det(m)<=0
      dp=0;
   end
end
```

c) Tomando los 2 primeros términos de la serie de Taylor del coseno:

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - .....$$

$$\frac{a}{1-\cos(x)} = \frac{a}{1-\left(1-\frac{x^2}{2!}\right)} = \frac{2a}{x^2}$$

d)

$$Tj = D^{-1}(L+U) = \begin{bmatrix} 0 & -1 \\ -2 & 0 \end{bmatrix} \qquad \rho(Tj) = \sqrt{2}$$

$$Tgs = (D-L)^{-1}U = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \qquad \rho(Tgs) = 2$$

Rpta.: $1 < \rho(Tj) < \rho(Tgs)$

e) tomando : $x = arcsen\left(e^{-x}\right) = g(x)$

$$g'(x) = \frac{-e^{-x}}{\sqrt{1-e^{-2x}}}$$

Tomando $x_0 = 0.5$

$$\left|g'(x_0)\right| = 0.76 < 1$$

El algoritmo será:

$$x_{n+1} = arcsen\left(e^{-x_{n+1}}\right) \quad n = 0, 1, ....$$

**Problema 2**

a)

*LU=A*

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ s & 0 & 1 \end{bmatrix}$$

Resolviendo:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ s/2 & -s & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1+s \end{bmatrix}$$

b)

$$U\ x_1 = L^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad x_1 = \begin{bmatrix} \frac{1}{s+1} \\ \frac{s-1}{s+1} \\ \frac{-s}{s+1} \end{bmatrix}$$

$$U\ x_2 = L^{-1}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad x_2 = \begin{bmatrix} \frac{-1}{s+1} \\ \frac{2}{s+1} \\ \frac{s}{s+1} \end{bmatrix}$$

$$U\ x_3 = L^{-1}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad x_1 = \begin{bmatrix} \frac{1}{s+1} \\ \frac{-2}{s+1} \\ \frac{1}{s+1} \end{bmatrix}$$

$$A^{-1} = X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \frac{1}{s+1}\begin{bmatrix} 1 & -1 & 1 \\ s-1 & 2 & -2 \\ -s & s & 1 \end{bmatrix}$$

c)

No es posible la factorizacion para $s = -1$

**Problema 3**

a)

$$M\ x'' + k\ x = 0$$
$$M\ x'' = -k\ x$$
$$x'' = -M^{-1}k\ x$$
$$A = M^{-1}k = \begin{bmatrix} 50 & -40 \\ -20 & 40 \end{bmatrix}$$

$$\lambda_1 = 45 + 5\sqrt{33} \qquad v_1 = \begin{bmatrix} 8 \\ 1-\sqrt{33} \end{bmatrix}$$
$$\lambda_2 = 45 - 5\sqrt{33} \qquad v_2 = \begin{bmatrix} 8 \\ 1+\sqrt{33} \end{bmatrix}$$

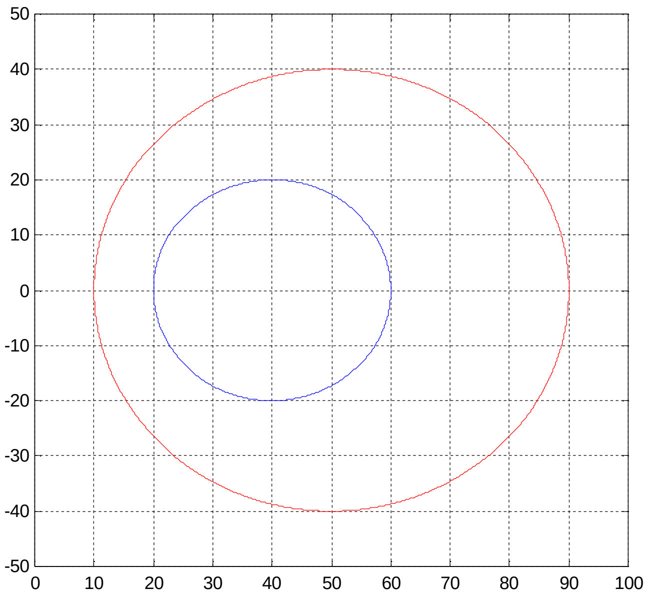
b)

Si es posible la diagonalizacion puesto que los valores propios son distintos y se puede obtener un vector propio para cada caso.

$$P = \begin{bmatrix} 8 & 8 \\ 1-\sqrt{33} & 1+\sqrt{33} \end{bmatrix} \qquad D = \begin{bmatrix} 45+5\sqrt{33} & 0 \\ 0 & 45-5\sqrt{33} \end{bmatrix} \qquad P^{-1}AP = D$$

c)

Aplicando Gershegorin: $\begin{cases} |z-50| \leq 40 \\ |z-40| \leq 20 \end{cases}$



d)

Potencia inversa con desplazamiento:

$$q = 90$$
$$B = (A - qI)^{-1}$$
$$B = \begin{bmatrix} -0.0417 & 0.0333 \\ 0.0167 & -0.0333 \end{bmatrix}$$
$$x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y_1 = Bx_0 = \begin{bmatrix} -0.0417 \\ 0.0167 \end{bmatrix} \quad u_1 = -0.0417 \quad x_1 = \begin{pmatrix} 1 \\ -0.4 \end{pmatrix} \quad \lambda_1 = q + \frac{1}{u_1} = 66$$

$$y_2 = Bx_1 = \begin{bmatrix} -0.0550 \\ 0.0300 \end{bmatrix} \quad u_2 = -0.0550 \quad x_2 = \begin{pmatrix} 1 \\ -0.5455 \end{pmatrix} \quad \lambda_2 = q + \frac{1}{u_2} = 71.8182$$

$$y_3 = Bx_2 = \begin{bmatrix} -0.0598 \\ 0.0348 \end{bmatrix} \quad u_3 = -0.0598 \quad x_3 = \begin{pmatrix} 1 \\ -0.5823 \end{pmatrix} \quad \lambda_3 = q + \frac{1}{u_3} = 73.2911$$

$$\varepsilon(\lambda) = |\lambda_3 - \lambda_2| = 1.4729$$

Converge al valor propio dominante 73.7228

**Problema 4**

a)

Biseccion

| a | b | x | err |
|---|---|---|---|
| 1 | 2 | 3 | 1 |
| 2 | 2.5 | 3 | 0.5 |
| 2 | 2.25 | 2.5 | 0.25 |
| 2 | 2.125 | 2.25 | 0.125 |

b) ¨

Newton-Raphson

$$f(x) = 0.1 - e^{-12/x} \cos\left(3\sqrt{x - x^2}\right)$$

$$f'(x) = e^{-12/x}\sin\left(3\sqrt{x-x^2}\right)\frac{3}{2\sqrt{x-x^2}}(1-2x) - e^{-12/x}\cos\left(3\sqrt{x-x^2}\right)\frac{12}{x^2}$$

$$x_o = 2.125$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\varepsilon(x) = \left|x_{n+1} - x_n\right|$$

| n | xn | error |
|---|---|---|
| 0 | 2.125 | ------ |
| 1 | 2.0472 | 0.0788 |
| 2 | 2.0255 | 0.0217 |
| 3 | 2.0241 | 0.0014 |

La convergencia es evidente !!